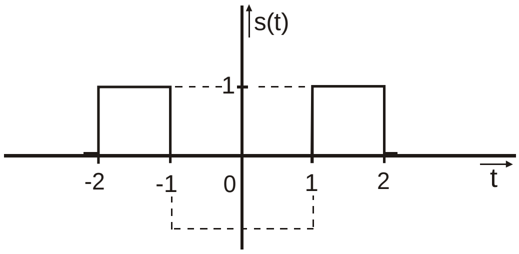
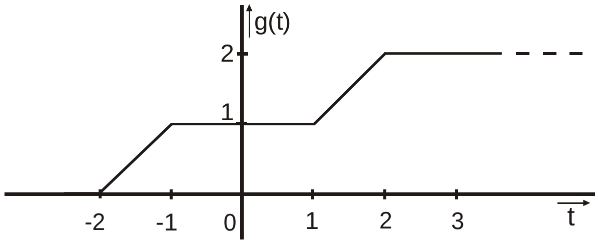
Aufgabe 1

1.1



1.2 $s'(t) = \delta(t+2) - \delta(t+1) + \delta(t-1) - \delta(t-2)$

1.3

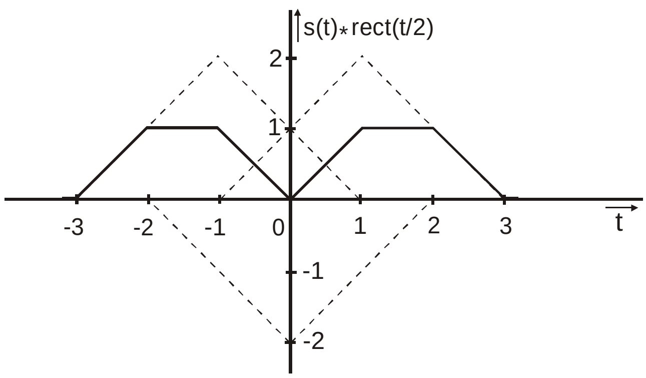


1.4 $\text{rect}\left(\frac{t}{2}\right) * h(t) \overset{!}{=} s(t) = \text{rect}\left(\frac{t}{2}\right) * \left[\delta(t+1) + \delta(t-1) - \delta(t)\right]$

$$h(t) = \delta(t+1) + \delta(t-1) - \delta(t)$$

1.5 $s(t) * \text{rect}\left(\frac{t}{2}\right) = \text{rect}\left(\frac{t}{2}\right) * \text{rect}\left(\frac{t}{2}\right) * \left[\delta(t+1) + \delta(t-1) - \delta(t)\right]$

$$= 2\Lambda\left(\frac{t}{2}\right) * \left[\delta(t+1) + \delta(t-1) - \delta(t)\right] = g(t) * \left[\delta(t+1) - \delta(t-1)\right]$$



1.6 $E = \int\limits_{-\infty}^{\infty} s^2(t)\, dt = 2$

1.7 $h_1(t) \neq 0$ für $t < 0 \Rightarrow$ nicht kausal

$$\int\limits_{-\infty}^{\infty} \left|h_1(t)\right| dt = 2 < \infty \Rightarrow \text{stabil}$$

Aufgabe 2

2.1

$$c_{\pm 1} = \frac{1}{2} e^{\pm j\varphi}; \quad \varphi = 0: \mathrm{Re}\{c_{\pm 1}\} = \frac{1}{2}; \mathrm{Im}\{c_{\pm 1}\} = 0$$

$$\varphi = \frac{\pi}{4}: \mathrm{Re}\{c_{\pm 1}\} = \frac{\sqrt{2}}{4}; \mathrm{Im}\{c_{\pm 1}\} = \pm \frac{j\sqrt{2}}{4} \qquad \varphi = \frac{\pi}{2}: \mathrm{Re}\{c_{\pm 1}\} = 0; \mathrm{Im}\{c_{\pm 1}\} = \pm \frac{j}{2}$$

2.2

$$S_1(j\omega) = \int_{-1/2}^{1/2} \cos(2\pi t) \cdot e^{-j\omega t} dt = \left[\frac{e^{-j\omega t}}{(2\pi)^2 - \omega^2} \left(-j\omega \cos(2\pi t) + 2\pi \sin(2\pi t) \right) \right]_{-1/2}^{1/2}$$

$$= \frac{j\omega}{(2\pi)^2 - \omega^2} \underbrace{\left[e^{-j\frac{\omega}{2}} - e^{j\frac{\omega}{2}} \right]}_{-2j\sin\frac{\omega}{2}} = \frac{2\omega}{(2\pi)^2 - \omega^2} \cdot \sin\left(\frac{\omega}{2}\right) = \left[\frac{1}{2} \left[\mathrm{si}\left(\pi - \frac{\omega}{2} \right) + \mathrm{si}\left(\pi + \frac{\omega}{2} \right) \right] \right]$$

2.3

$$t_0 = -\frac{\varphi T}{2\pi}$$

$$S_2(j\omega) = \left(\pi \left[\delta\left(\omega - \frac{2\pi}{T} \right) + \delta\left(\omega + \frac{2\pi}{T} \right) \right] \cdot e^{j\omega \frac{\varphi T}{2\pi}} \right) * \frac{1}{2\pi} T \,\mathrm{si}\left[\frac{\omega T}{2} \right]$$

$$= \frac{T}{2} \mathrm{si}\left[\frac{\omega T}{2} \right] * \left[\delta\left(\omega - \frac{2\pi}{T} \right) e^{j\varphi} + \delta\left(\omega + \frac{2\pi}{T} \right) e^{-j\varphi} \right]$$

$$= \frac{T}{2} e^{j\varphi} \cdot si\left(\frac{\omega T}{2} - \pi \right) + \frac{T}{2} e^{-j\varphi} si\left(\frac{\omega T}{2} + \pi \right)$$

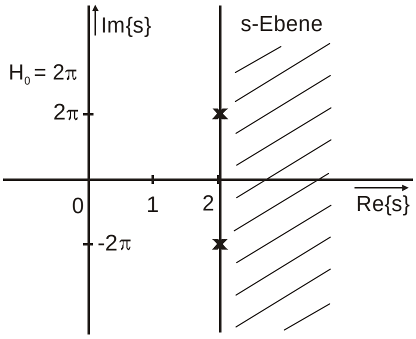
2.4

$$S_2(j\omega) = \left[\delta(\omega - 2\pi) + \delta(\omega + 2\pi) \right] * \frac{1}{2} \mathrm{si}\left[\frac{\omega}{2} \right]$$

$$= \frac{1}{2} \mathrm{si}\left[\frac{\omega - 2\pi}{2} \right] + \frac{1}{2} \mathrm{si}\left[\frac{\omega + 2\pi}{2} \right] = \frac{\sin\left(\frac{\omega}{2} - \pi \right)}{\omega - 2\pi} + \frac{\sin\left(\frac{\omega}{2} + \pi \right)}{\omega + 2\pi}$$

$$= -\sin\left(\frac{\omega}{2} \right) \cdot \frac{2\omega}{\omega^2 - (2\pi)^2} = \frac{2\omega}{(2\pi)^2 - \omega^2} \cdot \sin\left(\frac{\omega}{2} \right) = \frac{4}{(2\pi)^2 - \omega^2} \cdot si\left(\frac{\omega}{2} \right)$$

Aufgabe 3



3.1 $s^2 - 4s + 4 + (2\pi)^2 = 0$

$s_{p_{1,2}} = 2 \pm \sqrt{4 - 4 - (2\pi)^2} = 2 \pm j2\pi$

Konvergenzgebiet: $\text{Re}\{s\} > 2$

3.2 $F_1(s) = \dfrac{2\pi}{(s-2)^2 + (2\pi)^2} \overset{\mathcal{L}^{-1}}{\leftrightarrow} f_1(t) = e^{2t}\sin(2\pi t)\cdot\varepsilon(t)$

(nach Tabelle)

3.3 $f_2(t) = -\sin(2\pi t)e^{2t}\cdot\varepsilon(-t)$ (linksseitige Funktion)

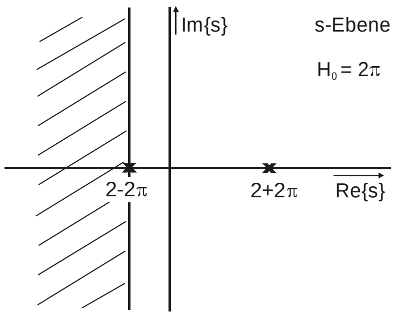
3.4 $F_2(s) = \dfrac{2\pi}{(s-2)^2 + (2\pi)^2} = F_1(s)$

Konvergenzgebiet: $\text{Re}\{s\} < 2$

3.5 $f_2(t) = -\sin(2\pi t)e^{-2t}\varepsilon(-t)$

$F_2(s) = \dfrac{2\pi}{(s+2)^2 + (2\pi)^2} \Rightarrow s_{p_{1,2}} = -2 \pm j2\pi$

Konvergenzgebiet: $\text{Re}\{s\} < -2$



3.6 $s^2 - 4s + 4 - (2\pi)^2 = 0$

$s_{p_{1,2}} = 2 \pm \sqrt{4 - 4 + (2\pi)^2} = 2 \pm 2\pi$

Konvergenzbereich: $\text{Re}\{s\} < +(2 - 2\pi)$

3.7 $F_3(s) = \dfrac{a_1}{s-(2+2\pi)} + \dfrac{a_2}{s-(2-2\pi)} = \dfrac{2\pi}{(s-2)^2 - (2\pi)^2}$

$\Rightarrow a_1 = \dfrac{1}{2} = -a_2$

$F_3(s) = \dfrac{\frac{1}{2}}{s-(2+2\pi)} - \dfrac{\frac{1}{2}}{s-(2-2\pi)}$; $\text{Re}\{s\} < 2 - 2\pi$

$f_3(t) = \dfrac{1}{2}\varepsilon(-t)\left[-e^{(2+2\pi)t} + e^{(2-2\pi)t}\right]$

$f_3(t) = -\varepsilon(-t)\cdot e^{2t}\cdot\sinh(2\pi t)$

Aufgabe 4

4.1 $$\left[a\,F(s)+b\,G(s)+c\cdot\frac{1}{s}G(s)\right]\cdot\frac{1}{s}=G(s)$$

$$G(s)\left[s-b-\frac{c}{s}\right]=a\,F(s)$$

$$H(s)=\frac{G(s)}{F(s)}=\frac{a\cdot s}{s^2-b\cdot s-c}$$

4.2 $$G(s)\left[s^2-bs-c\right]=a\,F(s)\cdot s$$

$$\Rightarrow\quad \frac{d^2}{dt^2}g(t)-b\frac{dg(t)}{dt}-cg(t)=a\frac{df(t)}{dt}$$

4.3 $$b=0\quad\Rightarrow\quad H(s)=\frac{as}{s^2-c}$$

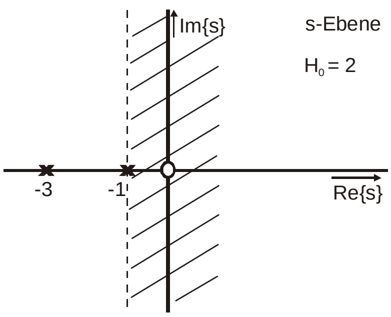
$$s_N=0\qquad s_{p_{1,2}}=\pm\sqrt{c}=\begin{cases}\pm\sqrt{c} & \text{für } c \text{ positiv}\\ \pm j\sqrt{|c|} & \text{für } c \text{ negativ}\end{cases}$$

Das System ist kausal;

Pole liegen nicht alle in der linken Hälfte der $s-$Ebene

$\Rightarrow$ System kann <u>nicht</u> stabil sein.

4.4 $$H_1(s)=\frac{2s}{s^2+4s+3}$$



$$s_N=0\qquad s_{p_{1,2}}=-2\pm\sqrt{4-3}=-2\pm1$$

a) rechtsseitiges Signal: $Kb:\mathrm{Re}\{s\}>-1$

$j\omega-$Achse liegt im $Kb\Rightarrow H(j\omega)$ existiert

b) linksseitiges Signal: $Kb:\ \mathrm{Re}\{s\}<-3\quad\Rightarrow H(j\omega)$ existiert nicht

c) zweiseitiges Signal: $Kb:\ -3<\mathrm{Re}\{s\}<-1\quad\Rightarrow H(j\omega)$ existiert nicht

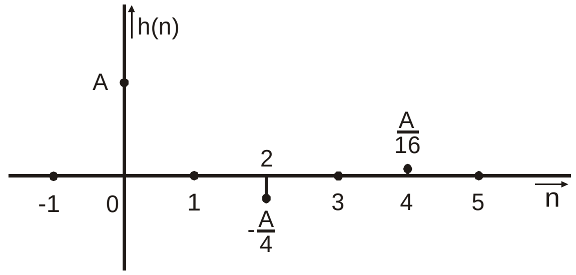
4.5 $$H_1(s)\ \overset{\mathcal{L}}{\leftrightarrow}\ h(t)\qquad h_\varepsilon(t)=h(t)*\varepsilon(t)$$

$$h_\varepsilon(t)\ \overset{\mathcal{L}}{\leftrightarrow}\ \frac{H_1(s)}{s}=\frac{2}{s^2+4s+3}=\frac{1}{s+1}-\frac{1}{s+3}$$

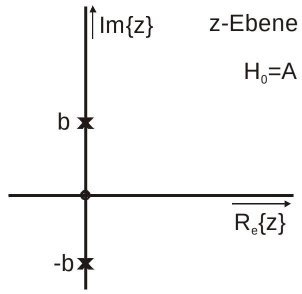
$$\Rightarrow\quad h_\varepsilon(t)=\varepsilon(t)\left[e^{-t}-e^{-3t}\right]$$

Aufgabe 5

5.1



5.2 Tabellenlösung mit $\Omega_1 = \frac{\pi}{2}$



$$H(z) = A\frac{1}{1+b^2z^{-2}} = A\frac{z^2}{z^2+b^2}$$

$$z_{N_{1,2}} = 0 \qquad z_{p_{1,2}} = \pm jb \qquad H_0 = A$$

Stabil, wenn $|b| < 1$,

d.h. Einheitskreis liegt im Konvergenzbereich

5.3 $b = \frac{1}{2} \quad z = e^{\sigma + j\Omega}$

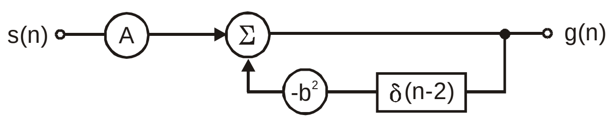
$$H(j\Omega) = A\frac{1}{1+b^2e^{-j2\Omega}} \qquad H(j\Omega)\big|_{\Omega=0} = \frac{A}{1+b^2} \overset{!}{=} 1$$

$$A = 1 + b^2 = \frac{5}{4}$$

5.4 $H(z) = \frac{G(z)}{S(z)} \quad \Rightarrow \quad G(z)\left(1+b^2z^{-2}\right) = A \cdot S(z)$

$$g(n) + b^2\, g(n-2) = A\, s(n) \quad \Rightarrow \quad g(n) = A\, s(n) - b^2\, g(n-2)$$

$Q = 0, \quad P = 2, \quad IIR -$ Filter 2. Ordnung



5.5 $H(z) \cdot H^I(z) = 1 \quad \Rightarrow \quad H^I(z) = \frac{1}{H(z)} = \frac{1+b^2z^{-2}}{A}$

$$h^I(n) = \frac{1}{A}\left[\delta(n) + b^2\,\delta(n-2)\right] \qquad Q = 2, P = 0, FIR -\text{Filter 2. Ordnung}$$

endliche Impulsantwort, stabil.