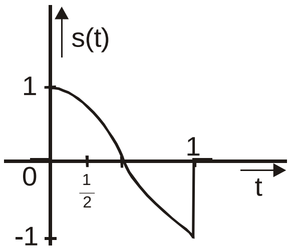
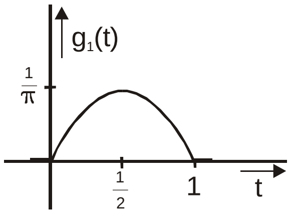
Musterlösung Aufgabe 1





1.1 $s(t)=\cos(\pi t)\operatorname{rect}\left(t-\frac{1}{2}\right)$

$$g_1(t)=\varepsilon(t)*s(t)=\int_{-\infty}^{t} s(\tau)\,d\tau$$

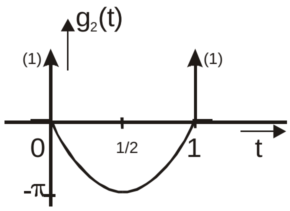
$$=\frac{1}{\pi}\sin(\pi t)\operatorname{rect}\left(t-\frac{1}{2}\right)$$

1.2 $E=\int_{-\infty}^{\infty} s^2(t)\,dt=\int_0^1 \cos^2(\pi t)\,dt$

$$=\frac{1}{2}\int_0^1\left[\cos(2\pi t)+1\right]dt=\frac{1}{2}$$

1.3 $g_2(t)=\frac{d}{dt}s(t)=-\pi\sin(\pi t)\operatorname{rect}\left(t-\frac{1}{2}\right)+\cos(\pi t)\left[\delta(t)-\delta(t-1)\right]$

$$=-\pi\sin(\pi t)\operatorname{rect}\left(t-\frac{1}{2}\right)+\delta(t)+\delta(t-1)$$

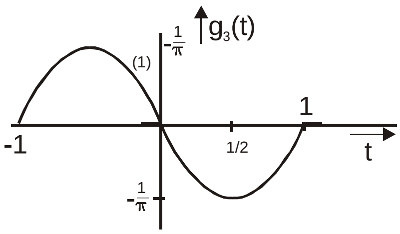


1.4 $g_3(t)=s(t)*\operatorname{rect}\left(t+\frac{1}{2}\right)=$

$$=s(t)*\left[\varepsilon(t+1)-\varepsilon(t)\right]=$$

$$=g_1(t+1)-g_1(t)$$

$$=\frac{1}{\pi}\sin\left(\pi(t+1)\right)\operatorname{rect}\left(\frac{t}{2}\right)$$



1.5 $Tr\left\{s(t)\right\}=g(t)=s(t)\cdot\cos(\omega_0 t)$

$$Tr\left\{\sum_i a_i s_i(t)\right\}=\left[\sum_i a_i s_i(t)\right]\cdot\cos(\omega_0 t)=\sum_i a_i g_i(t)\quad\Rightarrow\quad\text{linear}$$

$$Tr\left\{s(t-t_0)\right\}=s(t-t_0)\cdot\cos(\omega_0 t)$$

aber $\neq g(t-t_0)=s(t-t_0)\cdot\cos\left(\omega_0(t-t_0)\right)\quad\Rightarrow\quad$ nicht zeitinvariant

Musterlösung Aufgabe 2

2.1. Signal ist ungerade $s(-t) = -s(t) \rightarrow s_g(t) = 0$; $s_u(t) = s(t)$.

2.2.

$$s(t) = \mathrm{rect}\left(\frac{t}{T}\right) * \left[\delta(t - T_0) - \delta(t + T_0)\right]$$

$$\updownarrow \mathcal{F}$$

$$S(j\omega) = T \cdot \mathrm{si}\left(\frac{\omega T}{2}\right) . \left[e^{-j\omega T_0} - e^{j\omega T_0}\right] = -j2T\,\mathrm{si}\left(\frac{\omega T}{2}\right)\sin\left(\omega T_0\right)$$

$$\left|S(j\omega)\right| = 2T\left|\mathrm{si}\left(\frac{\omega T}{2}\right)\sin\left(\omega T_0\right)\right| \quad ; \quad \varphi(\omega) = \begin{cases} -\dfrac{\pi}{2}, \text{ wenn } \mathrm{si}\left(\dfrac{\omega T}{2}\right)\sin\left(\omega T_0\right) > 0 \\ \dfrac{\pi}{2}, \text{ wenn } \mathrm{si}\left(\dfrac{\omega T}{2}\right)\sin\left(\omega T_0\right) < 0 \end{cases}$$

alternative Lösung zu 2.2

$$S\left(j\omega\right) = -j\,2T \cdot \frac{2}{\omega T}\sin\left(\frac{\omega T}{2}\right)\sin\left(\omega T_0\right)$$

$$= j\frac{2}{\omega}\left[\cos\left(\omega\left(\frac{T}{2} + T_0\right)\right) - \cos\left(\omega\left(\frac{T}{2} - T_0\right)\right)\right]$$

2.3.

$$G_1(j\omega) = \frac{1}{2\pi}S(j\omega) * \left[\frac{\pi}{j}\delta\left(\omega - \omega_0\right) - \frac{\pi}{j}\delta\left(\omega + \omega_0\right)\right]$$

$$= T\left[\mathrm{si}\left(\frac{\left(\omega + \omega_0\right)T}{2}\right)\sin\left(\left(\omega + \omega_0\right)T_0\right) - \mathrm{si}\left(\frac{\left(\omega - \omega_0\right)T}{2}\right)\sin\left(\left(\omega - \omega_0\right)T_0\right)\right]$$

2.4. $\omega_0 T_0 = k \cdot \pi$ oder $\omega_0 T = l \cdot 2\pi$ (k, l ganzzahlig, $l \neq 0$)

2.5.

$$g_2(t) = \frac{1}{T}\mathrm{rect}\left(\frac{t}{T}\right) * \mathrm{rect}\left(\frac{t}{T}\right) * \left[\delta(t - T_0) - \delta(t + T_0)\right]$$

$$\updownarrow \mathcal{F}$$

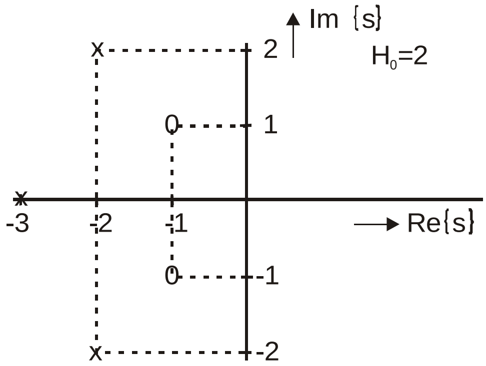
$$G_2(j\omega) = T \cdot \mathrm{si}^2\left(\frac{\omega T}{2}\right) . \left[e^{-j\omega T_0} - e^{j\omega T_0}\right] = -j2T\,\mathrm{si}^2\left(\frac{\omega T}{2}\right)\sin\left(\omega T_0\right)$$

Musterlösung Aufgabe 3

3.1 $h(t) = 2\varepsilon(t)\left[e^{-3t} - e^{-2t}\sin(2t)\right]$

$\overset{\mathcal{L}}{\leftrightarrow}$ $H(s) = 2\left[\dfrac{1}{s+3} - \dfrac{2}{(s+2)^2+4}\right]$ mit Formelsammlung

3.2 $H(s) = 2\cdot\dfrac{(s+2)^2+4-2s-6}{(s+3)\left((s+2)^2+4\right)} = 2\cdot\dfrac{s^2+2s+2}{(s+3)\left(s^2+4s+8\right)}$



Nullstellen: $s_{N_{1,2}} = -1 \pm j$

Pole: $s_{p_1} = -3$

$s_{p_{2,3}} = -2 \pm j2$

Konvergenzbereich: $\mathrm{Re}\{s\} > -2$

3.3 $H(j\omega) = H(s = j\omega)$, da $j\omega -$ Achse im Konvergenzbereich

3.4 $h_1(t) = h(t)\cdot e^{2t} \overset{\mathcal{L}}{\leftrightarrow} H_1(s) = H(s-2)$ (Formelsammlung)

$$H_1(s) = \frac{2}{s+1} - \frac{4}{s^2+4} = 2\cdot\frac{s^2-2s+2}{(s+1)\left(s^2+4\right)}$$

3.5 Pole und Nullstellen verschieben sich um $\mathrm{Re}\{s\} = 2$ nach <u>rechts!</u>

$s_{N_{1,2}} = 1 \pm j$, $s_{p_1} = -1$, $s_{p_{2,3}} = \pm j2$, $H_{1_0} = 2$

Konvergenzbereich: $\mathrm{Re}\{s\} > 0$

3.6 nicht stabil, da $j\omega -$ Achse nicht im Konvergenzbereich

bzw. $\int_0^\infty \left|h_1(t)\right| dt = 2\int_0^\infty \left|e^{-t} - \sin(2t)\right| dt = \infty$

Musterlösung Aufgabe 4

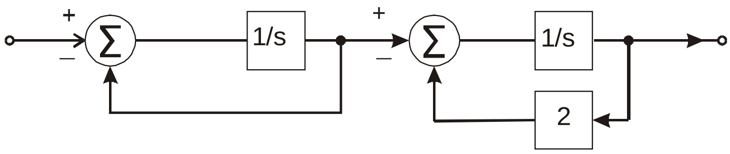
4.1 $$\frac{d^2 g(t)}{dt^2} + 3\frac{dg(t)}{dt} + 2g(t) = f(t)$$

$$\Rightarrow \quad s^2 \cdot G(s) + 3s \cdot G(s) + 2G(s) = F(s) \quad \Rightarrow \quad H(s) = \frac{G(s)}{F(s)} = \frac{1}{s^2+3s+2} = \frac{1}{s+1} \cdot \frac{1}{s+2}$$

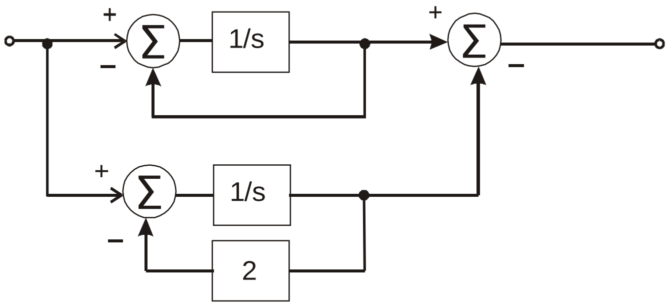
4.2 $$H(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

4.3 mit Integratoren:

a) Kaskadenschaltung (nach 4.1)

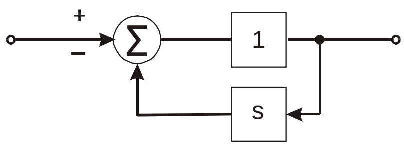


b) Parallelschaltung (nach 4.2)

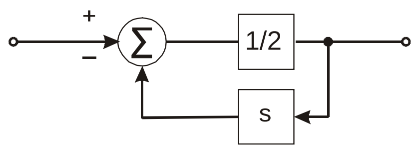


Mit Differentiatoren:

$$H_1(s) = \frac{1}{s+1}$$



$$H_2(s) = \frac{1}{s+2}$$

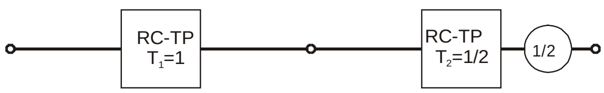


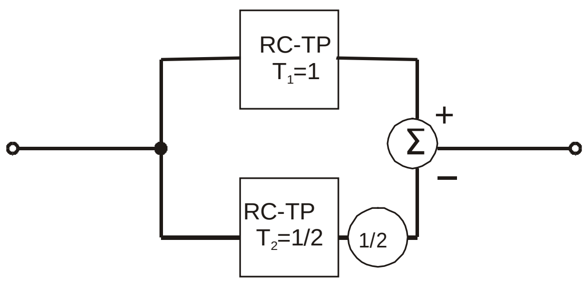
4.4 da kausal $\Rightarrow$ Konvergenzbereich: $\mathrm{Re}\{s\} > -1$

$\Rightarrow$ $j\omega$ – Achse im Konvergenzbereich $\Rightarrow$ $H(j\omega) = H(s = j\omega) = \frac{1}{1+j\omega} \cdot \frac{1}{2+j\omega}$

$$\Rightarrow \quad h(t) = \left[e^{-t}\varepsilon(t)\right] * \left[e^{-2t}\varepsilon(t)\right] = \left[\frac{1}{T_1} e^{-t/T_1}\varepsilon(t)\right] * \left[\frac{1}{T_2} e^{-t/T_2}\varepsilon(t)\right] \cdot T_1 \cdot T_2 \quad \Rightarrow \quad T_1 = 1 \;;\; T_2 = \frac{1}{2}$$

wegen $RC-Tiefpass$: $H(j\omega) = \frac{1}{1+j\omega RC} \leftrightarrow h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} \cdot \varepsilon(t)$ mit $R \cdot C = T$





Musterlösung Aufgabe 5

5.1.

$$H(z) = H_0 \frac{(z-1)^2}{z\left(z+\frac{1}{2}\right)} = H_0 \frac{1-2z^{-1}+z^{-2}}{1+\frac{1}{2}z^{-1}} \quad \text{für} \quad |z| > \frac{1}{2} \quad (\text{stabil wenn kausal})$$

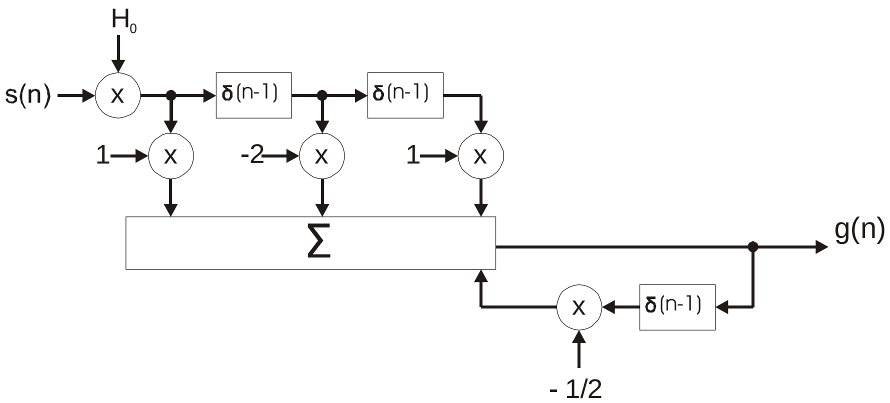
5.2. Nullstellen sperren bei Frequenz Ω=0; Polstelle bei z=–1/2 hebt Frequenz $\Omega=\pi$ an $\rightarrow$ Hochpass.

5.3.

$$H(j\Omega) = H_0 \frac{1-2e^{-j\Omega}+e^{-j2\Omega}}{1+\frac{1}{2}e^{-j\Omega}}$$

$$H(j\Omega)\Big|_{\Omega=\pi} = H_0 \frac{1+2+1}{1-\frac{1}{2}} = H_0 \cdot 8 \overset{!}{=} 1 \quad \Rightarrow \quad H_0 = \frac{1}{8}$$

5.4.



5.5.

$$H(z) = H_0 \left[\frac{1}{1-\left(-\frac{1}{2}\right)z^{-1}} - 2\frac{1}{1-\left(-\frac{1}{2}\right)z^{-1}} z^{-1} + \frac{1}{1-\left(-\frac{1}{2}\right)z^{-1}} z^{-2} \right]$$

$$\Rightarrow h(n) = H_0 \left[\varepsilon(n)\left(-\frac{1}{2}\right)^n - 2\varepsilon(n-1)\left(-\frac{1}{2}\right)^{n-1} + \varepsilon(n-2)\left(-\frac{1}{2}\right)^{n-2} \right]$$