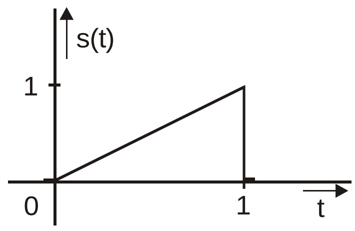
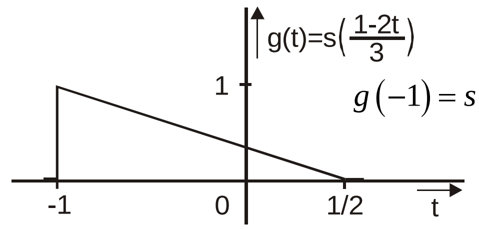
Musterlösung Aufgabe 2 GG ET IV

2.1



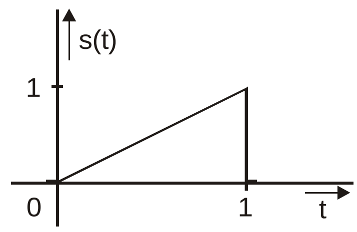


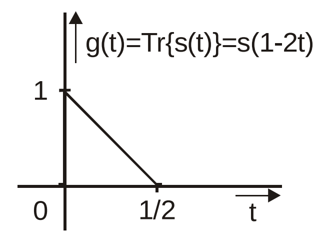
$g(-1) = s(1)$ und $g\left(\frac{1}{2}\right) = s(0)$

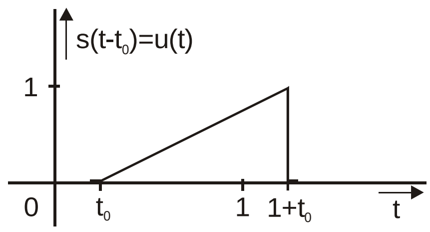
2.2 $s_i(t) \;\rightarrow\; g_i(t) = s_i(1-2t)$

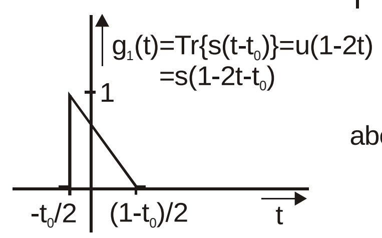
Linearität: $Tr\left\{\sum_i a_i s_i(t)\right\} = \sum_i a_i s_i(1-2t) = \sum_i a_i g_i(t) \;\Rightarrow$ lineares System

Zeitvarianz: Beispiel

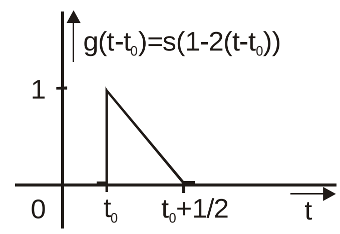






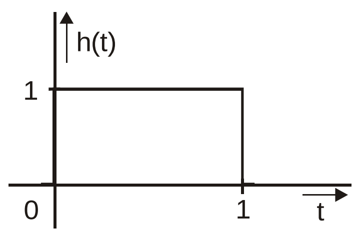


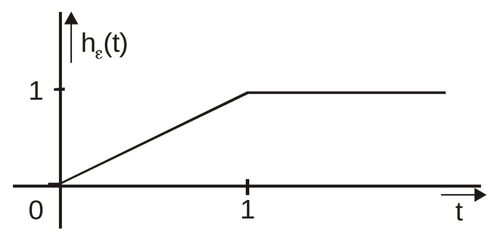
aber



$g_1\left(\frac{-t_0}{2}\right) = u(1+t_0)$ $\quad g_1\left(\frac{1-t_0}{2}\right) = u(t_0)$ $\quad g_1(t) \neq g(t-t_0) \Rightarrow$ nicht zeitvariant

2.3





$h(t) = \mathrm{rect}\left(\frac{t - 1/2}{1}\right)$ $\qquad h_\varepsilon(t) = t \cdot \varepsilon(t) - (t-1)\,\varepsilon(t-1)$

2.4 kausal, da $h(t) = 0$ für $t < 0$ amplitudenstabil, da $\int_0^\infty |h(t)|\,dt = 1 \quad < \infty$

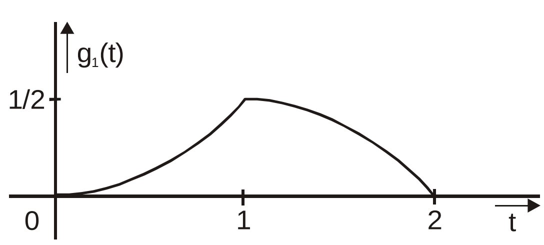
2.5 $g_1(t) = s(t) * h(t) = \int_{-\infty}^{\infty} s(\tau)\,h(t-\tau)\,d\tau$

$g_1(t) = 0$ für $t \leq 0$

$g_1(t) = \int_0^t \tau\,d\tau = \frac{1}{2}t^2$ für $0 \leq t \leq 1$

$g_1(t) = \int_{t-1}^{1} \tau\,d\tau = \frac{1}{2} - \frac{(t-1)^2}{2}$ für $1 \leq t \leq 2$

$g_1(t) = 0$ für $2 \leq t$



2.6 $\int_{-\infty}^{\infty} g_1(t)\,dt = \int_{-\infty}^{\infty} s(t)\,dt \cdot \int_{-\infty}^{\infty} h(t)\,dt = \frac{1}{2}$

Musterlösung Aufgabe 4 **GG ET IV**

4.1
$$S(j\omega)=\int_{-\infty}^{\infty}s(t)\cdot e^{-j\omega t}dt=\int_{-\infty}^{\infty}\frac{1}{T_1}\mathrm{rect}\left(\frac{t}{T_1}\right)e^{-j\omega t}dt=\frac{1}{T_1}\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}}e^{-j\omega t}dt=\frac{1}{-j\omega T_1}e^{-j\omega t}\bigg|_{-\frac{T_1}{2}}^{\frac{T_1}{2}}$$
$$=\frac{1}{-j\omega T_1}\left[e^{-\frac{j\omega T_1}{2}}-e^{+\frac{j\omega T_1}{2}}\right]=\frac{-2j}{-j\omega T_1}\sin\left(\omega\frac{T_1}{2}\right)=si\left(\omega\frac{T_1}{2}\right)$$

4.2
$$d_k=\frac{1}{T_2}\int_{-\frac{T_2}{2}}^{\frac{T_2}{2}}\left[\sum_{n=-\infty}^{\infty}\delta(t-nT_2)\right]e^{-jk\omega_0 t}dt=\frac{1}{T_2}\int_{-\frac{T_2}{2}}^{\frac{T_2}{2}}\delta(t-0\cdot T_2)e^{-j0\cdot\frac{2\pi}{T_2}t}dt=\frac{1}{T_2}\int_{-\frac{T_2}{2}}^{\frac{T_2}{2}}\delta(t)dt=\frac{1}{T_2},\quad \omega_0=\frac{2\pi}{T_2}$$

4.3
$$s_p(t)=s(t)*\delta_{T_2}(t)\quad ;\quad \omega_0=\frac{2\pi}{T_2}$$
$$\overset{\mathcal{F}}{\leftrightarrow}\; S_p(j\omega)=si\left(\omega\frac{T_1}{2}\right)\cdot\frac{2\pi}{T_2}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi}{T_2}k\right)=\frac{2\pi}{T_2}\sum_{k=-\infty}^{\infty}si\left(\frac{2\pi}{T_2}k\cdot\frac{T_1}{2}\right)\delta(\omega-k\omega_0)\overset{!}{=}2\pi\sum_{k=-\infty}^{\infty}c_k\delta(\omega-k\omega_0)$$
$$\Rightarrow c_k=\frac{1}{T_2}si\left(\frac{T_1}{T_2}\pi k\right)$$

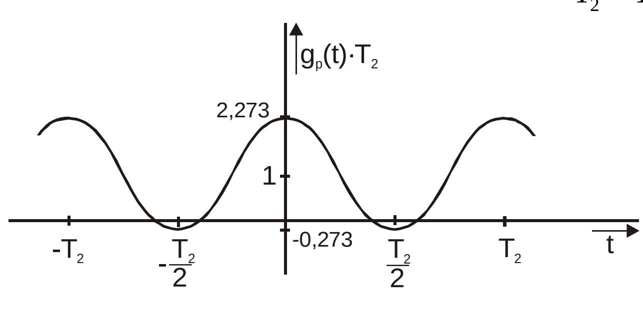
4.4
$$c_0=\frac{1}{T_2}si\left(\frac{T_1}{T_2}\pi\cdot 0\right)=\frac{1}{T_2}$$

4.5 Die Fourierreihe enthält Frequenzen $k\cdot\omega_0=k\cdot\frac{2\pi}{T_2}$

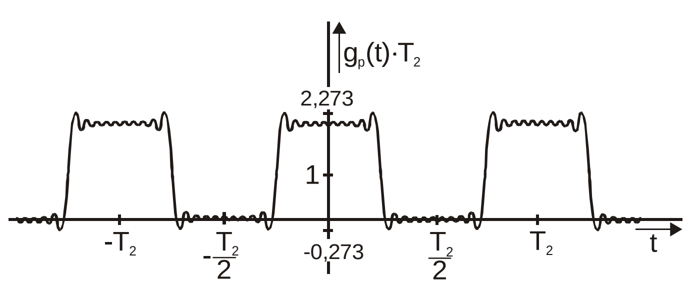
Nach Tiefpassfilterung bleiben nur die Frequenzen $|k\cdot\omega_0|\le\omega_g$ erhalten, also $k=\{-1;0;1\}$;

$c_0=\frac{1}{T_2}$; $c_1=c_{-1}=\frac{1}{T_2}si\left(\frac{\pi}{2}\right)=\frac{2}{T_2\pi}$ mit $T_1=\frac{T_2}{2}$

$$\Rightarrow g_p(t)=c_{-1}e^{-j\omega_0 t}+c_0+c_1e^{j\omega_0 t}=c_0+2c_1\cos(\omega_0 t)=\frac{1}{T_2}+\frac{4}{T_2\pi}\cos\left(2\pi\frac{t}{T_2}\right)$$



4.6 Es bleiben die Frequenzen $k\le 50$ übrig.



<u>Musterlösung Aufgabe 6</u> **GG ET IV**

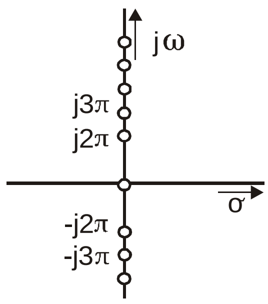
6.1 $$h(t) = \sin(\pi t)\,\mathrm{rect}\left(\frac{t-1}{2}\right) = \sin(\pi t)\cdot\varepsilon(t) - \sin\big(\pi(t-2)\big)\varepsilon(t-2)$$

$$\overset{\mathcal{L}}{\leftrightarrow} \quad H(s) = \frac{\pi}{s^2+\pi^2}\left[1-e^{-2s}\right]$$

6.2 Nullstellen: $e^{-2s} \overset{!}{=} 1 \;\Rightarrow\; \sigma = 0$ und $\omega_N = \pm n\cdot\pi \;\Rightarrow\; s_N = \pm jn\pi$

Polstellen: $s^2+\pi^2 \overset{!}{=} 0 \;\Rightarrow\; s_{p_{1,2}} = \pm j\pi$

$\Rightarrow$ die beiden Polstellen werden durch Nullstellen aufgehoben!

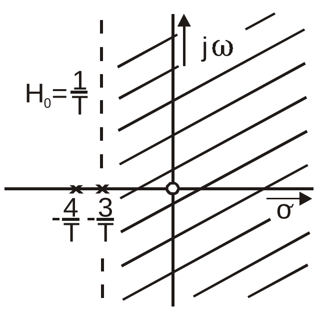


Konvergenzbereich: gesamte s – Ebene, da keine Polstellen!

Das System ist stabil, da $j\omega$ – Achse im Konvergenzbereich liegt, bzw. $h(t) =$ Energiesignal mit begrenzter Amplitude!

6.3 $$T^2\frac{d^2u_2(t)}{dt^2} + 7T\frac{du_2(t)}{dt} + 12u_2(t) = T\frac{du_1(t)}{dt}\Bigg| : T^2 \quad \overset{\mathcal{L}}{\leftrightarrow} \quad s^2U_2(s) + \frac{7}{T}sU_2(s) + \frac{12}{T^2}U_2(s) = \frac{1}{T}sU_1(s)$$

$$H_1(s) = \frac{U_2(s)}{U_1(s)} = \frac{1}{T}\cdot\frac{s}{s^2+\frac{7}{T}s+\frac{12}{T^2}} = \frac{1}{T}\frac{s}{\left(s+\frac{3}{T}\right)\left(s+\frac{4}{T}\right)}$$



Konvergenzbereich: $\mathrm{Re}\{s\} > -\frac{3}{T}$

6.4 $$U_2 = U_1(s)\cdot H_1(s) = \frac{U_0}{T^2}\cdot\frac{\left(1-e^{-sT}\right)}{s\left(s+\frac{3}{T}\right)\left(s+\frac{4}{T}\right)}$$

$$\frac{A}{s} + \frac{B}{s+\frac{3}{T}} + \frac{C}{s+\frac{4}{T}} \overset{!}{=} \frac{1}{s\left(s+\frac{3}{T}\right)\left(s+\frac{4}{T}\right)} \quad\Rightarrow\quad A = \frac{T^2}{12} \quad;\quad B = -\frac{T^2}{3} \quad;\quad C = \frac{T^2}{4}$$

$$\Rightarrow \quad U_2(s) = \frac{U_0}{12}\left(\frac{1}{s} - 4\frac{1}{s+\frac{3}{T}} + 3\frac{1}{s+\frac{4}{T}}\right)\left(1-e^{-sT}\right)$$

$$\overset{\mathcal{L}}{\leftrightarrow} \quad u_2(t) = \frac{U_0}{12}\left[\varepsilon(t)\left(1-4e^{-3\frac{t}{T}}+3e^{-4\frac{t}{T}}\right) + \varepsilon(t-T)\left(1-4e^{-3\frac{t-T}{T}}+3e^{-4\frac{t-T}{T}}\right)\right]$$

<u>Musterlösung Aufgabe 8</u> **GG ET IV**

8.1 Aus $R_i \to \infty$ folgt: $i_+ = i_- = 0$

wegen $u_2(t) < \infty$ und $\upsilon_0 \to \infty$ folgt $u_D = 0$

8.2 $U_0(s) = I_0(s) \cdot sL + U_1(s)$; mit $u_D = 0$

$$\Rightarrow \quad U_0(s) = I_0(s) \cdot sL + I_0(s) \cdot \frac{R}{2}$$

$$\Rightarrow \quad Z(s) = \frac{U_0(s)}{I_0(s)} = sL + \frac{R}{2}$$

8.3 Spannungsteiler zwischen $\frac{R}{2}$ und sL:

$$H_1(s) = \frac{R/2}{R/2 + sL} = \frac{R}{2L} \cdot \frac{1}{s + \frac{R}{2L}} = \frac{1}{\tau_1} \cdot \frac{1}{s + \frac{1}{\tau_1}} \quad \text{mit } \tau_1 = \frac{2L}{R}$$

Konvergenzbereich: $\sigma > -\frac{R}{2L} = -\frac{1}{\tau_1}$

8.4 Operationsverstärker in invertierender Beschaltung:

$$H_2(s) = -\frac{1}{sRC} = -\frac{1}{s\tau_2} \qquad \text{mit } \tau_2 = R \cdot C$$

Konvergenzbereich: $\sigma > 0$

8.5
$$H(s) = H_1(s) \cdot H_2(s) = -\frac{1}{s(2sLC + RC)} = -\frac{1}{\tau_1 \tau_2} \cdot \frac{1}{s\left(s + \frac{1}{\tau_1}\right)}$$

Konvergenzbereich: $\sigma > 0$

8.6
$$H(s) = -\frac{1}{RC}\left(\frac{1}{s} - \frac{1}{s + \frac{R}{2L}}\right) = -\frac{1}{\tau_2}\left(\frac{1}{s} - \frac{1}{s + \frac{1}{\tau_1}}\right)$$

$$\Rightarrow \quad h(t) = \frac{1}{RC}\varepsilon(t)\left(e^{-\frac{R}{2L}t} - 1\right) = -\frac{1}{\tau_2}\varepsilon(t)\left(1 - e^{-t/\tau_1}\right)$$

8.7 $H(s)$ hat einfachen Pol im Ursprung, damit kann die Fourier-Transformierte von $h(t)$ nicht als $H(s = j\omega)$ gebildet werden.

$$h(t) = -\frac{1}{\tau_2}\varepsilon(t) \cdot \left(1 - e^{-t/\tau_1}\right)$$

$$\overset{\mathcal{F}}{\leftrightarrow} \quad H(j\omega) = -\frac{1}{\tau_2}\left(\pi\delta(\omega) + \frac{1}{j\omega} - \frac{1}{j\omega + \frac{1}{\tau_1}}\right) = -\frac{1}{RC}\left(\pi\delta(\omega) + \frac{1}{j\omega} - \frac{1}{\frac{R}{2L} + j\omega}\right)$$

Musterlösung Aufgabe 10 **GG ET IV**

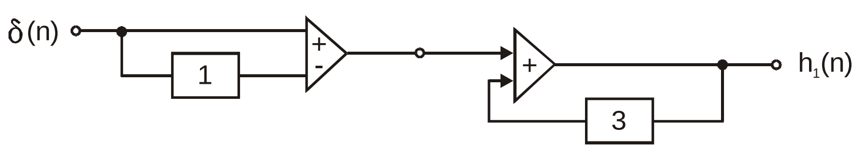
10.1 $$H_1(z) = \frac{z^2}{\left(z+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\left(z+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)} = \frac{z^2}{z^2+z+1} = \frac{1}{z^{-2}+z^{-1}+1}$$

10.2 Das Filter ist nicht stabil, da beide Pole nicht innerhalb des Einheitskreises der z – Ebene liegen:

$$\left|z_{P_{1,2}}\right| = \left|-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right| = 1$$

10.3 $$G(z)\left(1+z^{-1}+z^{-2}\right) = S(z)$$

$$\overset{z}{\leftrightarrow} g(n)+g(n-1)+g(n-2) = s(n)$$
$$\Leftrightarrow g(n) = s(n)-g(n-1)-g(n-2)$$
$$\Leftrightarrow h_1(n) = \delta(n)-h_1(n-1)-h_1(n-2)$$
$$= \delta(n)-\left(\delta(n-1)-h_1(n-2)-h_1(n-3)\right)-h_1(n-2)$$
$$= \delta(n)-\delta(n-1)+h_1(n-3)$$



$$h_1(n) = \sum_{k=0}^{\infty} \delta(n-3k)-\delta(n-3k-1)$$

10.4 $$H_2(z) = 1+z^{-1}+z^{-2}$$

10.5 $$h_3(n) = h_1(n) * h_2(n) \overset{z}{\leftrightarrow} H_3(z) = H_1(z)H_2(z) = \frac{1+z^{-1}+z^{-2}}{1+z^{-1}+z^{-2}} = 1 \quad \overset{z}{\leftrightarrow} \quad h_3(n) = \delta(n)$$

10.6 $$H_2(j\Omega) = 1+e^{-j\Omega}+e^{-j2\Omega} = e^{-j\Omega}\left(e^{j\Omega}+1+e^{-j\Omega}\right) = e^{-j\Omega}\left(1+2\cos\Omega\right)$$

$$\Rightarrow \left|H_2(j\Omega)\right| = \left|1+2\cos\Omega\right|$$

$$\varphi(\Omega) = -\Omega \pm k\cdot\pi \qquad \text{mit} \qquad k = \begin{cases} 1 & \textit{für}\ \cos(\Omega)\left(1+2\cos(\Omega)\right) < 0 \\ 0 & \textit{sonst} \end{cases}$$