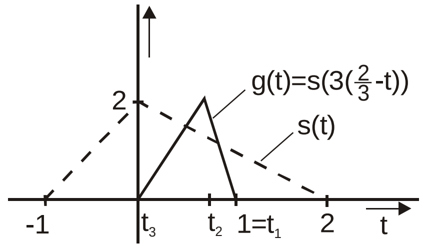
Musterlösung Aufgabe 1

1.1 $g(t_1) = s(2-3t_1) \overset{!}{=} s(-1) \quad \Rightarrow t_1 = 1$

$g(t_2) = s(2-3t_2) \overset{!}{=} s(0) \quad \Rightarrow t_2 = \frac{2}{3}$

$g(t_3) = s(2-3t_3) \overset{!}{=} s(2) \quad \Rightarrow t_3 = 0$



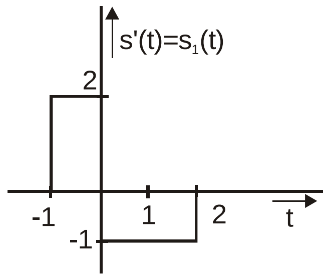
1.2 $Tr\left\{\sum_i a_i s_i(t)\right\} = \sum_i a_i Tr\{s_i(t)\} = \sum_i a_i s_i(2-3t) = \sum_i a_i g_i(t) \Rightarrow$ linear

$Tr\{s(t-t_0)\} = s(2-t_0-3t) \neq g(t-t_0) = s(2-3(t-t_0)) \quad \Rightarrow$ nicht zeitinvariant

1.3 $s_1(t) = s'(t)$

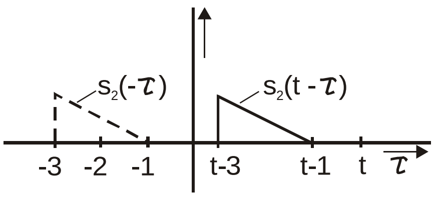
$\Rightarrow \quad s'(t) * \varepsilon(t) = s(t)$

$s_1(t) = 2\,\mathrm{rect}\left(t+\frac{1}{2}\right) - \mathrm{rect}\left(\frac{t-1}{2}\right)$



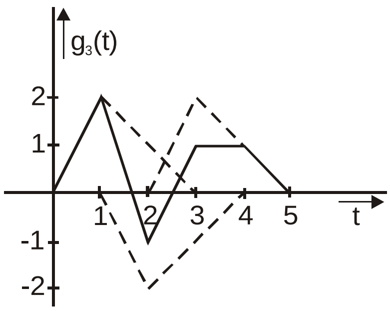
1.4 $g_2(t) = s(t) * s_2(t) = \int_{-\infty}^{\infty} s(\tau) s_2(t-\tau)\, d\tau$

$g_2(t) = 0 \begin{cases} \textit{für} & t < 0 \\ \textit{und} & t > 5 \end{cases}$



1.5 $h(t) = \delta(t-1) - \delta(t-2) + \delta(t-3)$

$g_3(t) = s(t-1) - s(t-2) + s(t-3)$



1.6 $\sin(2\pi t) * \left[\varepsilon(t) e^{-t}\right] = \int_{-\infty}^{\infty} \sin(2\pi\tau) e^{-(t-\tau)} \varepsilon(t-\tau)\, d\tau$

$$= e^{-t} \int_{-\infty}^{t} \sin(2\pi\tau) e^{\tau} d\tau = e^{-t} \cdot \left[\frac{e^{\tau}}{1+4\pi^2}\left(\sin(2\pi\tau) - 2\pi\cos(2\pi\tau)\right)\right]_{-\infty}^{t}$$

$$= \frac{1}{1+4\pi^2}\left[\sin(2\pi t) - 2\pi\cos(2\pi t)\right]$$

Musterlösung Aufgabe 2

2.1 $s(t) = a\,\mathrm{rect}\left(\frac{t}{2T_1}\right) * \sum_{n=-\infty}^{\infty} \delta(t - nT)$

$S(j\omega) = 2aT_1 \quad si(\omega T_1) \cdot \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$

$= 4\pi a \frac{T_1}{T} \sum_{k=-\infty}^{\infty} si\left(\frac{2\pi k}{T} \cdot T_1\right) \quad \delta\left(\omega - \frac{2\pi k}{T}\right) \overset{!}{=} 2\pi \sum_{k=-\infty}^{\infty} c_k \delta(\omega - k\omega_0)$

$\Rightarrow \quad s(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$ mit $c_k = 2a \cdot \frac{T_1}{T} si\left(2\pi \frac{T_1}{T} k\right)$

und $\omega_0 = \frac{2\pi}{T}$

2.2 $s(t - t_0) \Rightarrow c_k^1 = c_k e^{-jk\omega_0 t_0} = c_k\left[\cos(k\omega_0 t_0) - j\sin(k\omega_0 t_0)\right]$

2.3 $P = \frac{1}{T}\int_{-T_1}^{T_1} a^2 dt = \frac{a^2}{T} 2 \cdot T_1 = a^2 \cdot T_1 \cdot \frac{2}{T} = \frac{4V^2}{2\Omega} = 2W$

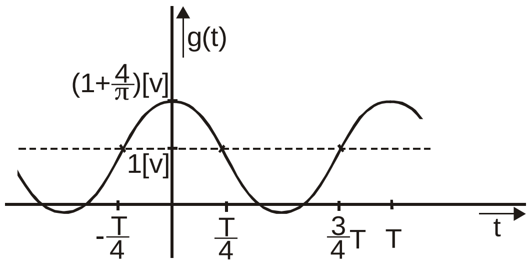
$a_1^2 \cdot 2\frac{T_2}{2} = a^2 \cdot 2\frac{T_1}{T} \quad \Rightarrow \quad a_1^2 = a^2 \cdot \frac{T_1}{T_2} \quad \Rightarrow \quad a_1 = a\sqrt{\frac{T_1}{T_2}}$

2.4 $H(j\omega) = \mathrm{rect}\left(\frac{\omega T}{8\pi}\right) \overset{!}{=} \mathrm{rect}\left(\frac{\omega}{2\omega_g}\right) \quad \Rightarrow \quad \omega_g = \frac{4\pi}{T} = 2\omega_0$

$G(j\omega) = 4\pi a \cdot \frac{T_1}{T} \sum_{k=-2}^{2} si\left(2\pi \cdot k \frac{T_1}{T}\right) \delta\left(\omega - \frac{2\pi}{T} \cdot k\right) = 4\pi a \frac{T_1}{T} \sum_{k=-2}^{2} si(k\omega_0 T_1)\, \delta(\omega - k\omega_0)$

$c_k = 2a\frac{T_1}{T} si(k\omega_0 T_1) = 1V si\left(k \cdot \frac{\pi}{2}\right) \quad \Rightarrow \quad c_0 = 1V \quad ; \quad c_{\pm 1} = \frac{2}{\pi}V \quad ; \quad c_{\pm 2} = 0$

$g(t) = c_0 + 2c_1 \cos(\omega_0 t) = 1V + \frac{4}{\pi}V\cos(\omega_0 t)$



2.5 $P_g = \sum_{k=-\infty}^{\infty} |c_k|^2 = c_0^2 + 2c_1^2 = \left(1 + \frac{8}{\pi^2}\right)W$

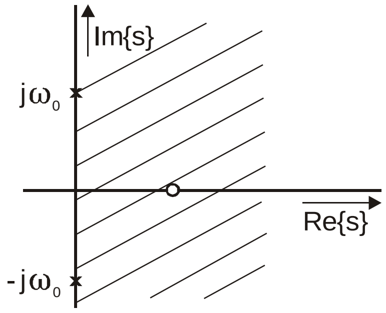
Musterlösung Aufgabe 3

3.1 $f(t)=\cos(\omega_0 t+\varphi)\cdot\varepsilon(t)=\frac{1}{2}\left[e^{j(\omega_0 t+\varphi)}+e^{-j(\omega_0 t+\varphi)}\right]\cdot\varepsilon(t)$

$$=\frac{1}{2}e^{j\varphi}\left[e^{j\omega_0 t}\varepsilon(t)\right]+\frac{1}{2}e^{-j\varphi}\left[e^{-j\omega_0 t}\varepsilon(t)\right]=\cos(\omega_0 t)\cdot\varepsilon(t)\cdot\cos(\varphi)-\sin(\omega_0 t)\cdot\varepsilon(t)\sin\varphi$$

$$\overset{\mathcal{L}}{\leftrightarrow}\quad F(s)=\frac{s\cdot\cos(\varphi)}{s^2+\omega_0^2}-\frac{\omega_0\cdot\sin(\varphi)}{s^2+\omega_0^2}=\frac{s\cos(\varphi)-\omega_0\sin\varphi}{s^2+\omega_0^2}$$

3.2 $s_{p1,2}=\pm j\omega_0$ $\qquad s_N=\omega_0\,tg(\varphi)$



Konvergenzgebiet: $\mathrm{Re}\{s\}>0$, (da kausales Signal)

3.3 $H(s)=\frac{s^2-s+1}{s^2(s-1)}=\frac{A}{s-1}+\frac{B}{s}+\frac{C}{s^2}=\frac{s^2(A+B)+(C-B)s-C}{s^2(s-1)}$

$$\Rightarrow\quad A=1\quad;\quad B=0\quad;\quad C=-1$$

$$\Rightarrow\quad H(s)=\underbrace{\frac{1}{s-1}}_{\mathrm{Re}\{s<1\}}-\underbrace{\frac{1}{s^2}}_{\mathrm{Re}\{s>0\}}\quad\overset{\mathcal{L}^{-1}}{\leftrightarrow}\quad h(t)=-e^t\cdot e(-t)-t\,\varepsilon(t)$$

3.4 zweiseitiges Signal $\Rightarrow$ nicht kausal

a) rechtsseitiger Anteil: Pol bei $\sigma=0$ $\Rightarrow$ nicht stabil!

b) linksseitiger Anteil: Pol bei $\sigma=1$ $\Rightarrow$ stabil

3.5 $H(s)=\frac{U_2(s)}{U_1(s)}\quad\Rightarrow\quad U_2(s)\cdot\left[s^3-s^2\right]=U_1(s)\cdot\left[s^2-s+1\right]$

$$\Rightarrow\quad\frac{d^3u_2(t)}{dt^3}-\frac{d^2u_2(t)}{dt^2}=\frac{d^2u_1(t)}{dt^2}-\frac{d\,u_1(t)}{dt}+u_1(t)$$

Musterlösung Aufgabe 4

4.1 $$H_A(s) = \left(H_1(s) + H_2(s)\right) \cdot \frac{H_1(s)}{1 + H_1(s) \cdot H_2(s)}$$

4.2 id. Integrator: $H_1(s) = \frac{1}{s}$ id. Differentiator: $H_2(s) = s$

$$\Rightarrow \quad H_A(s) = \left(\frac{1}{s} + s\right) \cdot \frac{1}{s} \; \frac{1}{1+1} = \frac{1+s^2}{2 \cdot s^2}$$

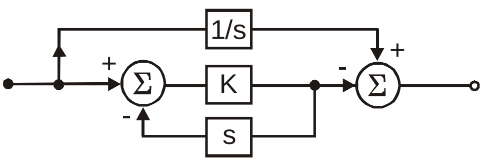
4.3 Polstellen: $S_{p1,2} = 0 \quad \Rightarrow \quad j\omega$ – Achse liegt nicht im Konvergenzbereich

$\Rightarrow$ das System ist nicht stabil.

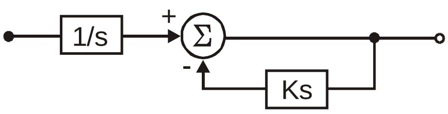
4.4 $$h(t) = \varepsilon(t)\left(1 - e^{-\frac{t}{k}}\right)$$

$$\overset{\mathcal{L}}{\leftrightarrow} \quad H(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{k}} = \frac{\frac{1}{k}}{s\left(s + \frac{1}{k}\right)}$$

4.5 a) $H(s) = \frac{1}{s} - \frac{k}{1 + sk}$



oder b) $H(s) = \frac{1}{s} \cdot \frac{1}{1 + ks}$



Musterlösung Aufgabe 5

5.1 $S_r(j\omega) = S_a(j\omega) \cdot H(j\omega)$

$$= \left\{ S(j\omega) * \left[\frac{1}{2\pi} \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right) \right] \right\} \cdot \frac{1}{T} T \, si\left(\frac{\omega T}{2}\right) \cdot T si\left(\frac{\omega T}{2}\right)$$

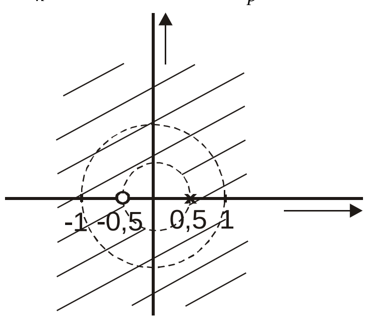
$$= \sum_{k=-\infty}^{\infty} S\left(j\left(\omega - \frac{2\pi k}{T}\right)\right) \cdot si^2\left(\frac{\omega T}{2}\right)$$

5.2 $S(j\omega) \overset{!}{=} S_r(j\omega) \cdot H_r(j\omega) = S_a(j\omega) \cdot H(j\omega) \cdot H_r(j\omega)$

$$\Rightarrow \quad H_r(j\omega) = \frac{T}{H(j\omega)} \cdot \mathrm{rect}\left(\frac{\omega}{2\omega_g}\right) = \frac{1}{si^2\left(\frac{\omega T}{2}\right)} \, \mathrm{rect}\left(\frac{\omega}{2\omega_g}\right)$$

5.3 $H(z) = \dfrac{z + 0,5}{z - 0,5}$

$z_n = -0,5 \qquad z_p = 0,5$



Kausales System: $Kb: |z| > 0,5$

Pole liegen innerhalb des Einheitskreises!

Das System ist stabil.

5.4 $H(z) = \dfrac{1 + 0,5z^{-1}}{1 - 0,5z^{-1}} = \dfrac{1}{1 - 0,5z^{-1}} + 0,5z^{-1} \cdot \dfrac{1}{1 - 0,5z^{-1}}$

$$h(n) = \left(\frac{1}{2}\right)^n \varepsilon(n) + \frac{1}{2} \cdot \left(\frac{1}{2}\right)^n \varepsilon(n) * \delta(n-1)$$

$$h(n) = \left(\frac{1}{2}\right)^n \left[\varepsilon(n) + \varepsilon(n-1)\right]$$

$$H(z) = \frac{G(z)}{S(z)} = \frac{1 + 0,5z^{-1}}{1 - 0,5z^{-1}} = \frac{\sum_{q=0}^{Q} a_q z^{-q}}{1 - \sum_{p=1}^{P} b_p z^{-p}}$$

$\Rightarrow \quad a_0 = 1 \quad ; \quad a_1 = 0,5 \quad ; \quad b_1 = 0,5$

mit $g(n) = \sum_{q=0}^{Q} a_q s(n-q) + \sum_{p=1}^{P} b_p \, g(n-p)$

$\Rightarrow \quad g(n) = s(n) + 0,5\, s(n-1) + 0,5 g(n-1)$