<u>Musterlösung Aufgabe 1</u>

<u>1.1</u>

$$g(t) = \frac{d}{dt} s(t) = s(t) * \delta'(t)$$

$h(t) = \delta'(t) \mathrel{\hat{=}}$ idealer Differentiator ist ein LTI – System

<u>1.2</u>

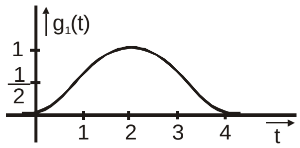
$g_1(t) = 0$ für $t < 0$

$g_1(t) = \int_0^t \tau d\tau = \frac{1}{2} t^2$ für $0 \le t \le 1$

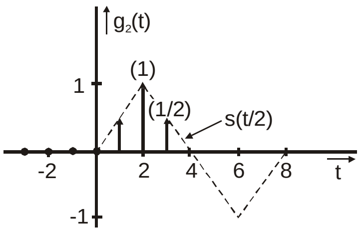
$g_1(t) = \frac{1}{2} + \int_1^t (2-\tau)\, d\tau = \frac{1}{2} + \left[2\tau - \frac{\tau^2}{2} \right]_1^t = 2t - \frac{t^2}{2} - 1$ für $1 \le t \le 3$

$g_1(t) = \frac{1}{2} + \int_3^t (\tau - 4)\, d\tau = \frac{1}{2} + \left[\frac{\tau^2}{2} - 4\tau \right]_3^t = 8 - 4t + \frac{t^2}{2}$ für $3 \le t \le 4$

$g_1(t) = 0$ für $t > 4$



<u>1.3</u>



$$g_2(t) = \frac{1}{2}\delta(t-1) + \delta(t-2) + \frac{1}{2}\delta(t-3)$$

<u>1.4</u> $\int_{-\infty}^{\infty} g_3(t) dt = \int_{-\infty}^{\infty} s(t)\, dt \cdot \int_{-\infty}^{\infty} g(t)\, dt = 0 \cdot 0 = 0$

<u>1.5</u> $g(t) = Tr\{s(t)\} = s(t) \cdot \cos(2\pi f_0 t)$

$Tr\left\{ \sum_i a_i s_i(t) \right\} = \left[\sum_i a_i s_i(t) \right] \cdot \cos(2\pi f_0 t) = \sum_i a_i g_i(t) \Rightarrow$ Linear!

$Tr\{s(t-t_0)\} = s(t-t_0) \cdot \cos(2\pi f_0 t)$

$g(t-t_0) = s(t-t_0) \cos(2\pi f_0 (t-t_0)) \ne Tr\{s(t-t_0)\} \Rightarrow$ nicht zeitinvariant

Aufgabe 2

2.1

$$c_0 = \frac{1}{T}\int_0^T t^2 dt = \frac{1}{T}\left[\frac{t^3}{3}\right]_0^T = \frac{T^2}{3}$$

$$c_k = \frac{1}{T}\int_0^T t^2 e^{-j\frac{2\pi k}{T}t} dt = \frac{e^{-j\frac{2\pi k}{T}t}}{T}\left[\frac{Tt^2}{-j2\pi k} - \frac{2T^2 t}{-4\pi^2 k^2} + \frac{2T^3}{j8\pi^3 k^3}\right]_0^T$$

$$= T^2\left[\frac{1}{2\pi^2 k^2} + j\frac{1}{2\pi k}\right]$$

2.2

$$c_k^{(1)} = c_k + c_{-k} e^{-jk\cdot 2\pi} = c_k + c_k^* = 2\cdot\mathrm{Re}\{c_k\} \Rightarrow c_0^{(1)} = \frac{2T^2}{3} \quad ; \quad c_k^{(1)} = \frac{T^2}{\pi^2 k^2}$$

$$c_k^{(2)} = c_k - c_{-k} e^{-jk\cdot 2\pi} = c_k - c_k^* = j2\,\mathrm{Im}\{c_k\} \Rightarrow c_0^{(2)} = 0 \quad ; \quad c_k^{(2)} = j\frac{T^2}{\pi k}$$

2.3

$$P = \sum_{k=-\infty}^{\infty}\left|c_k^{(1)}\right|^2 = \left(\frac{2}{3}\right)^2 + \frac{2}{\pi^4}\sum_{k=1}^{\infty}\frac{1}{k^4} = \frac{4}{9} + \frac{1}{45} = \frac{7}{15}$$

2.4

$$g_1(t) = \frac{2}{3} + \frac{2}{\pi^2}\cos 2\pi t$$

Musterlösung Aufgabe 3

3.1 $$H(s) = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-st} dt = \frac{1}{-s} e^{-st} \Big|_{-\frac{1}{2}}^{\frac{1}{2}} = -\frac{1}{s}\left[e^{-\frac{s}{2}} - e^{\frac{s}{2}} \right]$$

$$= \frac{e^{\frac{s}{2}} - e^{-\frac{s}{2}}}{s} = \frac{1}{s}\sinh\left(\frac{s}{2}\right)$$

3.2 $s_p = 0$

Nullstellen: $e^{\frac{s}{2}} \stackrel{!}{=} e^{-\frac{s}{2}} \quad \Rightarrow \quad e^{\frac{\sigma}{2}} e^{j\frac{\omega}{2}} \stackrel{!}{=} e^{-\frac{\sigma}{2}} e^{-j\frac{\omega}{2}}$

$\Rightarrow \quad \sigma = 0$ und $\cos\left(\frac{\omega}{2}\right) + j\sin\left(\frac{\omega}{2}\right) \stackrel{!}{=} \cos\left(\frac{\omega}{2}\right) - j\sin\left(\frac{\omega}{2}\right)$

$\Rightarrow \quad \frac{\omega}{2} = k \cdot \pi \quad \Rightarrow \quad s_{Nk} = j2k\pi$

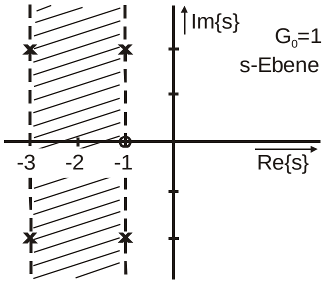
Pol und Nullstelle bei $s = 0$ heben sich auf.

$\Rightarrow$ Konvergenzbereich: ganze s – Ebene.

3.3 $$G(s) = \frac{2}{s^2 + 6s + 13} \quad + \quad \frac{s+1}{s^2 + 2s + 5}$$

$s^2 + 6s + 13 = 0 \qquad s^2 + 2s + 5 = 0$

$s_{p1,2} = -3 \pm j2 \qquad s_{p3,4} = -1 \pm j2 \qquad s_N = -1$



3.4 $$G(s) = \frac{2}{(s+3-j2)(s+3+j2)} + \frac{s+1}{(s+1-j2)(s+1+j2)}$$

wegen $(x+jy)(x-jy) = x^2 + y^2$

$$\Rightarrow \quad G(s) = \frac{2}{(s+3)^2 + 4} \quad + \quad \frac{s+1}{(s+1)^2 + 4}$$

$\mathrm{Re}\{s\} > -3 \qquad \mathrm{Re}\{s\} < -1$

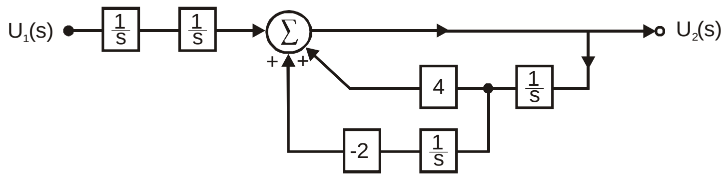
mit Tabelle:

$$g(t) = e^{-3t}\sin(2t)\cdot\varepsilon(t) - e^{-t}\cos(2t)\varepsilon(-t)$$

Musterlösung Aufgabe 4

4.1
$$\frac{d^2}{dt^2}u_2(t)-4\frac{d}{dt}u_2(t)+2u_2(t)\,dt=u_1(t)$$
$$s^2U_2(s)-4sU_2(s)+2U_2(s)=U_1(s)$$
$$H(s)=\frac{U_2(s)}{U_1(s)}=\frac{1}{s^2-4s+2}$$

4.2
$$U_2(s)=\frac{1}{s^2}U_1(s)+\frac{4}{s}U_2(s)-\frac{2}{s^2}U_2(s)$$



4.3
$$F(s)-a\,G(s)-b\,s\,G(s)=s^2G(s)$$
$$F(s)=G(s)\left[s^2+b\,s+a\right]$$
$$H_2(s)=\frac{G(s)}{F(s)}=\frac{1}{s^2+b\,s+a}$$

4.4
$$H_3(s)=\frac{1}{s^2+ds+4}\qquad s_{p1,2}=-\frac{d}{2}\pm\sqrt{\frac{d^2}{4}-4}$$

stabil, wenn alle Pole in der linken Hälfte der s – Ebene liegen bei kausalen Systemen

Es muss: $\mathrm{Re}\{s_{p1,2}\}\overset{!}{<}0$ sein!

i) reelle Pole: $\frac{d^2}{4}-4\geq 0$ und $-\frac{d}{2}\pm\sqrt{\frac{d^2}{4}-4}\overset{!}{<}0$

$\Rightarrow\ d^2\geq 16$ und $\frac{d}{2}>\pm\sqrt{\frac{d^2}{4}-4}$ $\Rightarrow$ $\underline{d\geq 4}$

ii) komplexe Pole: $\frac{d^2}{4}-4<0$ und $\underline{d>0}$ $\Rightarrow$ $\underline{d<4}$

aus i) und ii) folgt: $d>0$

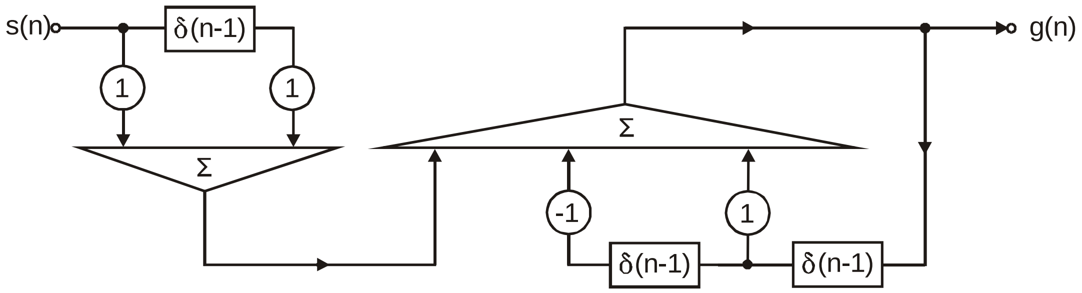
Aufgabe 5

5.1

$$h(0)=1;\ h(1)=1+1=2;\ h(2)=2-1=1;\ h(3)=1-2=-1;\ h(4)=-1-1=-2$$
$$h(5)=-2+1=-1;\ h(6)=-1+2=1;\ h(7)=1+1=2;\ h(8)=2-1=1$$

5.2

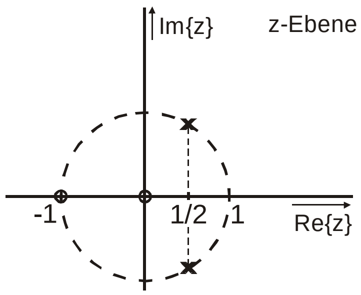
Skizze ...



5.3

$$G(z)\left[1-z^{-1}+z^{-2}\right]=S(z)\left[1+z^{-1}\right]\Rightarrow H(z)=\frac{1+z^{-1}}{1-z^{-1}+z^{-2}}=\frac{z(z+1)}{z^2-z+1}$$

$$z_{N_1}=0;\quad z_{N_2}=-1;\quad z_{p1/2=}\frac{1}{2}\pm\sqrt{\frac{1}{4}-1}=\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$$



Pole auf Einheitskreis $\left|z_p\right|^2=1$ (Winkel $\pm 60^0$): instabil

5.4

$$S(j\Omega)=2\pi c\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$$

$$G(j\Omega)=S(j\Omega)\cdot H(j\Omega)\quad\text{mit}\quad z^{-n}=e^{-jn\Omega}\quad\Rightarrow\quad H(j\Omega)=\frac{1+2e^{-j\Omega}+2e^{-j2\Omega}}{1-0{,}8e^{-j\Omega}+0{,}8\cdot e^{-j2\Omega}}$$

Wegen Siebeigenschaft:

$$\Rightarrow\quad G(j\Omega)=2\pi c\sum_{k=-\infty}^{\infty}\frac{1+2e^{-jk2\pi}+2e^{-jk\cdot 4\pi}}{1-0{,}8e^{-jk\cdot 2\pi}+0{,}8\cdot e^{-jk\cdot 4\pi}}\,\delta(\Omega-k\cdot 2\pi)$$

$$=2\pi c\cdot 5\sum_{k=-\infty}^{\infty}\delta(\Omega-k\cdot 2\pi)$$

$$\overset{\mathcal{F}}{\leftrightarrow}\quad g(n)=5\cdot c\sum_{k=-\infty}^{\infty}\delta(n-k)=5c$$